A Determination of Maximum Temperature Using Fourier Analysis

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Abstract: The Fourier analysis of Maximum temperature determination was carried out reference to three selected Nigerian Meteorological Stations (NIMET) data. Weather forecast of parameters such as Temperature, relative humidity and so on, are taking from the NIMET station and can be easily determined using Fourier analysis method. The modeling of a periodic time series using Fourier analysis method was employed to forecast future values of one of the weather parameters. The Maximum temperature data of three selected stations across; Sokoto (13°04’N 5°14’E), Maiduguri (11°50’N 13°09’E), and Ilorin (8°30’N 4°33’E), were analyzed using Fourier Twelve Point Analysis. The results showed that the alternative method has been proffered in the determination of weather temperature. The Maximum temperature was observed to be at the highest between months of March and July (23.49°C-25.85°C) for Ilorin, (26.17-26.50°C) for Maiduguri and Sokoto (26.15-26.30°C) respectively for both observed and estimated values and the lowest Maximum temperature is around December and January. It was concluded that Fourier analysis method can be used to forecast future values of temperature.

Keywords: Fourier Analysis, Modeling, NIMET, Weather forecast.

1.0 Introduction

Fourier analysis is the study of the way general function may be represented and approximated by sums of simpler trigonometric functions. Fourier analysis grew from the study of Fourier series and is named after Joseph Fourier, who showed that representing a function greatly simplifies the study of heat transfer. Fourier analysis encompasses a vast spectrum of mathematics including science and engineering. The process of decomposing a function into smaller pieces is often called Fourier analysis (Stroud K.A, 1996).

Nigeria like other countries experiences changes in temperature. The temperature varies monthly and is different for the various states in the country. Aweda et al., (2017) report that at high temperature there will be more evaporation. Allen and Smith (1994) reported that direct solar radiation and the ambient temperature of the air provide the necessary energy for evaporation. The atmospheric weather parameters are of great importance in the study of the atmosphere. Barry and Chorley (1976), reported that weather is generally considered as the state of the atmosphere at a given time at any given location. It may also be referred to as the aspects of the atmospheric state which is visible and experienced and which affect human activities (Ogolo and Adeyemi 2009). Being able to predict this change; it can help a country plan ahead. For example, farmers can plan when to plant certain crops so has to achieve adequate sunshine and rainfall for the crop. Also being able to predict this change helps to plan some season dependent projects adequately. Fourier analysis described earlier is the method used in this paper to analyse the temperature data. In this paper it will be determined that Fourier analysis fits well with the data and is a very efficient method to use in predicting future temperature data values. The French applied mathematician, Joseph Fourier worked on the methods of partial differential equations in Fourier series.

These factors initiate and influence the atmospheric processes (Ayoade, 1993). Rim (2004) reported that solar radiation was the most sensitive meteorological factor affecting evaporation while
wind speed was the least sensitive factor. Adeyemi and Aro (2004) reported that the variations of surface water vapor density in four Nigerian stations showed that surface water vapor density is higher at night by an average of 9.9% than during the day in the southern stations while in the midland station of Minna, the reverse is the case.

2.0 Materials and Method

For the modeling of the Fourier series analysis Atmospheric parameter data were collected from the Nigerian Meteorological Agency (NIMET). The data were filtered so as to remove the unwanted once from it. The parameter used was Maximum Temperature which spread across ten years (2001-2010). The averages of data were taking and the results were analyzed using Fourier series method for twelve point.

2.1 Fourier series

The Fourier series is represented by the following expressions:

\[ f(x) = a_0 + \sum_{n=1}^{12} \left( a_n \cos \frac{2\pi}{l} + b_n \sin \frac{2\pi}{l} \right) \]

Where,

1. \[ a_0 = \frac{1}{L} \int_0^L f(x) dx = 2 \times \text{mean value of } f(x) \text{ over a period} \]
2. \[ a_n = \frac{1}{L} \int_0^L f(x) \cos nx \ dx = 2 \times \text{mean value of } f(x) \text{cosnx over a period} \]
3. \[ b_n = \frac{1}{L} \int_0^L f(x) \sin nx \ dx = 2 \times \text{mean value of } f(x) \text{sinnx over a period} \]

\( n \) = number of variables, \( l \) = Latitude of the station or city or place in reference.

2.2 Twelve Point Analysis

In practice, one complete cycle of the function is divided into 12 equal strips, i.e. at interval of \( \frac{2\pi}{12} = \frac{\pi}{6} = 30^\circ \) and the ordinates \( y_n \) are tabulated. Remember that the final boundary ordinate \( y_n \) is omitted since this is regarded as the first ordinate of the next cycle.

From the data collected it would be observed that some examples such as a year consist of 12 months; therefore, a year represents a complete cycle. Values of \( f(x) \) for temperature, a period function \( 2\pi \). At interval \( 30^\circ \) from \( x = 30^\circ \) to \( x = 360^\circ \) are given for a year. Values of \( f(x) \) temperature, a period function \( 2\pi \). At interval \( 30^\circ \) from \( x = 30^\circ \) to \( x = 360^\circ \) are given for a year.

Table 2.1: Showing the twelve-point analysis

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
<th>330</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(x)</td>
<td>32.7</td>
<td>33.4</td>
<td>35.2</td>
<td>31.8</td>
<td>32.3</td>
<td>31.6</td>
<td>28.2</td>
<td>28.1</td>
<td>28.4</td>
<td>28.6</td>
<td>31.3</td>
<td>32.03</td>
</tr>
</tbody>
</table>

Now \[ a_0 = 2 \times \text{mean value of } f(x) \text{ over a cycle} \]

\[ a_0 = \frac{1}{12} \sum_{1}^{12} y_n = \frac{1}{12} (y_0 + y_1 + y_2 + \ldots + y_{11}) = a_0 \]

Now \[ a_n = 2 \times \text{mean value of } f(x) \text{cosnx over a period} \]

\[ a_1 = 2 \times \text{mean value of } f(x) \text{cosx over a period} \]
\[ a_2 = 2 \times \text{mean value of } f(x) \text{cos2x over a period} \]
\[ a_n = 2 \times \text{mean value of } f(x) \text{cosnx over a period} \]

Now \[ b_n = 2 \times \text{mean value of } f(x) \text{sinnx over a period} \]

\[ b_1 = 2 \times \text{mean value of } f(x) \text{sinx over a period} \]
\[ b_2 = 2 \times \text{mean value of } f(x) \text{sin2x over a period} \]

These are the cosine coefficient,
\[ b_n = 2 \cdot \text{mean value of } f(x) \sin nx \text{ over a period} \]

These are the sine coefficient of the period function.

Using the 12-point analysis, the data collected were analyzed, and the results were arranged as follows:

After tabulation of the corresponding Fourier sine and cosine coefficients.

\[
a_n = \frac{1}{\pi} \sum f(x) \cos nx \\
b_n = \frac{1}{\pi} \sum f(x) \sin nx
\]

Where \(a_n\) and \(b_n\) the corresponding amplitudes and phase angles are computed using the following formulas 7 and 8 stated above.

**Table 2.2:** Sine and Cosine Coefficients of the periodic function of each station

<table>
<thead>
<tr>
<th>Station</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(b_1)</th>
<th>(b_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sokoto</td>
<td>44.745</td>
<td>0.587</td>
<td>2.787</td>
<td>-0.006</td>
<td>-2.948</td>
<td>0.338</td>
</tr>
<tr>
<td>Maiduguri</td>
<td>-40.347</td>
<td>0.8</td>
<td>2.019</td>
<td>-0.125</td>
<td>-1.855</td>
<td>0.282</td>
</tr>
<tr>
<td>Ilorin</td>
<td>44.03</td>
<td>-0.448</td>
<td>2.774</td>
<td>0.76</td>
<td>-4.913</td>
<td>1.401</td>
</tr>
</tbody>
</table>

**Table 2.3:** Average Maximum Temperature Data for Sokoto 2001-2010

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>17.44</td>
<td>19.28</td>
<td>23.61</td>
<td>26.50</td>
<td>26.77</td>
<td>25.14</td>
<td>23.48</td>
<td>22.92</td>
<td>23.06</td>
<td>22.84</td>
<td>20.01</td>
<td>17.32</td>
</tr>
</tbody>
</table>

**Fourier equation for Sokoto station**

\[ f(x) = (1/2) \cdot (44.745 + 0.587 \cos x + 2.787 \cos 2x + (-0.006) \cos 3x + (-2.948) \sin x + 0.338 \sin 2x + 1.003 \sin 3x) \]

Where, equations 9 and 10 are used to generate the values on table 2.3 above. Therefore, other stations follow suit.

**Table 2.4:** Square of Sine and Cosine Coefficients of the periodic function of each station

<table>
<thead>
<tr>
<th>Station</th>
<th>(a_1^2)</th>
<th>(a_2^2)</th>
<th>(a_3^2)</th>
<th>(b_1^2)</th>
<th>(b_2^2)</th>
<th>(b_3^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sokoto</td>
<td>0.345</td>
<td>7.768</td>
<td>0.000036</td>
<td>8.69</td>
<td>0.114</td>
<td>1.006</td>
</tr>
<tr>
<td>Maiduguri</td>
<td>0.64</td>
<td>4.076</td>
<td>0.016</td>
<td>3.441</td>
<td>0.079</td>
<td>0.412</td>
</tr>
<tr>
<td>Ilorin</td>
<td>0.2</td>
<td>7.695</td>
<td>0.58</td>
<td>24.14</td>
<td>1.963</td>
<td>0.929</td>
</tr>
</tbody>
</table>
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\[ A_n = \sqrt{(a_n^2 + b_n^2)} \]

Phase Angle
\[ \theta_n = \tan^{-1} \frac{a_n}{b_n} \]

The results of the Amplitudes and phase angles are tabulated thus:

Phase Angle
\[ \phi_n = \tan^{-1} \frac{a_n}{b_n} \]

The results of the Amplitudes and phase angles are tabulated thus:

Table 2.5: Amplitude and Phase angles of the stations

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( \phi_1 )</th>
<th>( \phi_2 )</th>
<th>( \phi_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sokoto</td>
<td>0.5866</td>
<td>2.7873</td>
<td>0.0057</td>
<td>-0.2</td>
<td>1.45</td>
<td>-0.01</td>
</tr>
<tr>
<td>Maiduguri</td>
<td>0.8001</td>
<td>2.018</td>
<td>0.1246</td>
<td>-0.41</td>
<td>1.43</td>
<td>-0.19</td>
</tr>
<tr>
<td>Ilorin</td>
<td>0.4484</td>
<td>2.7739</td>
<td>0.76</td>
<td>0.09</td>
<td>1.1</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Fig. 2.1a: Estimated value of Sokoto

Fig. 2.1b: Observed value of Sokoto
Fig. 2.1c: Estimated and Observed values of Maximum temperature for Sokoto station

Fig. 2.2a: Estimated value of Ilorin

Fig. 2.2b: Observed value of Ilorin

Fig. 2.2c: Estimated and Observed values of Maximum temperature of Ilorin
3.0 DISCUSSION

Figure 2.1a and 2.1b shows the point at which the Maximum temperature is highest around April and May in Sokoto. While the point at which the Maximum temperature is lowest around January and December for the observed and the estimated values. At this period, the atmosphere is cloudier which could lead to drop in temperature giving raise to harmattan and dew. This could be attributed to solar radiation and some atmospheric parameters such as cloud, wind speed and some other parameters that can affect the decrease of the Maximum temperature. Figure 2.2a and 2.2b shows the temperature pattern for Ilorin in terms of their estimated and observed values. There is a slight shape increase in the figure 2.2b which could be as a result equipment malfunction during the period. Meanwhile, Ilorin also have is highest Maximum temperature in the month of May. Maiduguri a Northern state of Nigeria could have is highest Maximum temperature in the month of May and June during the peak of the wet season. These buttress the fact from figure 2.3 above where the higher temperature was observed to be around May and June. More so, distance from sea locations could affect the rise the Maximum temperature. Maiduguri and
Sokoto are both very far from the sea while Ilorin is closer to the sea. From figure 2.3 above, it shows that the point at which the Maximum temperature is highest is around May while the point at which the Maximum temperature is lowest is at January and December. It is observed that the point at which the Maximum temperature is generally highest is between the months of March and June while it is generally lower between the months of December and January. This can be as a result of human activities and increase in solar radiation. The point at which the Maximum temperature is highest is around March while the point at which the Maximum temperature is lowest is at January.

4.0 CONCLUSION
This paper has proffered an alternative forecast solution on the measurement of weather parameters. From the three stations considered it could be observed that both the estimated and the observed are values are much closed. This implies that the method used is accurate in the determination of weather parameters.

REFERENCES


